Main model

**Introduction**

In order to discover the influence of size, shape and merging pattern, we propose a two-dimensional Cellular Automata (CA) model. Our model is based on the one-dimensional *Nagel Schreckenberg* model, which was first presented in 1992 and successfully demonstrated many features of the traffic flow. Compared with a one-dimensional CA model, a two-dimensional one is more complex but feasible enough to simulate the real traffic flow. Therefore, the results from a two dimensional CA model is relatively more accurate. Basically, the CA model can be regarded as an effective method to simulate features of traffic jams by showing how interactions between nearby vehicles cause the deceleration.

**Assumptions**

* We assume that all the drivers are selfish and short-sighted. To be specific, drivers always adopt measures to move to the Roads Leading to Exit (RLE), wherever they are.
* We assume that once a vehicle leaves the exit of the toll plaza, it will accelerate, namely no congestion outside the plaza.
* We assume that both the choices towards tollbooths and the coming time of vehicles satisfies random distribution.

司机都是自私和短视的，在宽广的departure zone 上，他们总是采取尽可能向前行驶和向主路靠的措施

车辆一旦驶出plaza就会加速离开，plaza之外没有拥堵

所有的车都是规格相同的小轿车，不考虑大型客车或货车的影响

车对于收费亭的选择都是随机的，它们到来的时间也是随机分布的

**Model Establishing**

In our model, the toll plaza is divided into cells. In our two-dimensional CA model, each cell is evenly distributed in the shape of square. So the toll plaza, or rather, the whole departure area consists of plenty of square cells. which are exactly the same as the cells are evenly distributed in a particular shape of the square. An independent cell or an adjacent cell cluster can denote both empty roads or vehicles according to their corresponding sizes, such as length and width. is not an empty road is covered by the car, a car according to their size can occupy multiple cells. We can assign a certain speed to each cell-formed vehicle. But each speed value must be an integer, ranging from 0 to an identified v\_max. Each vehicle is assigned a speed, which is an integer from 0 to Vmax.

In addition, time is also discretized. Each time step is defined as the time that a car takes to travel past the length of 10 cars at the speed of the restricted value. Time is also discretized to time steps. Between two time steps\During a step interval, vehicles are set to perform the following actions sent by corresponding instructions in order. At each time step,the following actions are conducted in order from first to last and all are applied to all cars. Another important rule is that vehicles always perform their updated actions at the same time. In each action the updates are applied to all cars in parallel.

Several actions are further explained as follows:

* **Acceleration:** It reflects a characteristic that vehicles tends to travel as fast as possible. Here, this action obeys a rule as:

if v\_t<v\_max, then v -> min(v\_t+1,vmax)

* **Deceleration:** This action guarantee no collision with the vehicle ahead. It satisfies:

V\_t -> min(v\_t, dx)

* **Randomization：**It embodies behavioral discrepancy of drivers. The introduction of randomization not only reflects random accelerative actions, but also reflects the overreaction in decelerative processes. In conclusion, it is a key factor that causes congestion. Likewise, it meets following principle:

If v\_t > 0, then v\_t ->v\_t-1 with probability *p\_v*

* **Steering:** It describes the trends that divers are more willing to turn to the Roads Leading to Exit (RLE) if they are not on the RLE at this moment. The corresponding rule is:

If (y>WL), then v\_y=1 with probability p\_y

* **Lane changing:** When a vehicle gets stuck, the driver is likely to convert his or her lane to a nearby one (only the one closer to the RLE) if that lane is empty. It is also set to satisty:

If dy>1 and v=0, then vy=1 with probability p\_d

* **Horizontal velocity:**

V\_x=sqrt(v^2 - vy^2）

* **Motion:** It indicates that **t**he position of a vehicle is shifted by its speed v\_x and v\_y, thus, x-> x+dx, y ->y+dy
* **Incoming vehicles:** Each vehicle will travel from the start of a certain tollbooth with the probability of p\_in. This probability can denote traffic density in some ways.

Where,

V\_t The speed of current moment

V\_max The maximum speed allowed

V\_t+1 The speed of next moment

dx The nearest distance between a vehicle being observed and another in its horizontal direction.

dy The nearest distance between a vehicle being observed and another in its vertical direction.

p\_v The probability of randomlization

p\_y The probability of sheering

p\_d The probability of lane changing

p\_in The probability of incoming vehicles

v\_x The horizontal velocity

v\_y The vertical velocity

w\_l The exit width of the toll plaza

**Brief Results**

We convert our thoughts and design above into program instructions via Python, and get some interesting results. Figure 1 shows free flow, while Figure 2 describes crowded flow.

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We can conclude from the simulations above that the RLE are the most crowded part once traffic density increases to cause congestion. In this case, drivers always consider turning to the RLE as soon as possible, which causes unevenly distribution of traffic density in the departure zone.

In the following chapters, we are primarily discussing more detailed results and exploring the relations between the toll plaza and a variety of those important indicators.